# THE MOUNDS OF CYDONIA - A CASE STUDY FOR PLANETARY SETI

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The Society for Planetary SETI Research (SPSR) has as its aim the study of features on planetary surfaces, to evaluate possible signs of ET activity in the form of landscape modifications or other alterations not easily attributable to natural geological formation. This paper displays one such study, based in part on a previous one which showed that a group of twelve mound-like formations in the Cydonia area of Mars, of relatively small and nearly uniform size, have relative positions that repeatedly display symmetries well beyond chance. It focuses primarily on five of those mounds, showing that they display some unusual and precise geometrical features highlighted by close connections to sequences of prime numbers. This paper also reviews the related statistical anomaly found in the relative placement of these mounds and discusses some recent critiques of that work. Previous work showed that the frequency of appearance of related right and isosceles triangles in the mound distribution cluster sharply in density about a certain value of the angle defining those related triangles. In order to assess the role of the special angle itself as a possible source in the sharp clustering in the density of appearances of these triangles, this paper reports a new statistical study that confirms the extent to which the favored and redundant geometry implied by that angle has a self-duplication property. It involves an examination of each of 1 million randomly generated sets of 12 mounds with the same analysis techniques used for the actual Cydonia mounds. It is found that this property can account for only a small portion of the statistical anomaly found in the earlier work. Proposed further work includes an examination of a clear distinction between the 5 and 12 mound configurations and its possible relation to mound shapes using recent MGS high resolution images. Finally a discussion is given of a repeated connection between the triangles that appear in the ideal geometry of the 5 mound configuration and the basic quantum mechanics of spin angular momentum.

Keywords: Mounds, Cydonia, planetary SETI, geometry, prime numbers

## 1. INTRODUCTION

This paper describes a class of features investigated with the assistance of members of the Society for Planetary SETI Research (SPSR). The material is based in part on a paper published in the Journal of Scientific Exploration [1] and coauthored with SPSR founder, Professor Stanley V. McDaniel. It serves as a case study for the development of techniques in the search for signs of extraterrestrial intelligence on planetary surfaces. Among various proposals regarding possible media for ETI communication it has been argued that mathematics and geometry could provide a basis for establishing a common frame of reference for communication. It is reasonable, therefore, to examine the surfaces of terrestrial (or "rocky") planets in the solar system for signs of intelligent intervention, to give consideration to any indications of unusual geometry, and to develop techniques for the evaluation of such phenomena, should any appear.

In the following, some exploratory work is described that reveals remarkable geometric and fundamental mathematical relationships among certain features on the surface of Mars. These would appear to call for explanation and could be interpreted as evidence for intelligent intervention. Additionally a number of attempts at more prosaic interpretations are described.

## 2. GEOMETRY OF THE PENTAD OF MOUNDS

Figure 1 is from the 1976 Viking Satellite (image number

35A72) and shows the classic early morning image of the socalled "Face on Mars" (upper right). In the left side of the image are a number of surface features of roughly the same size. The objects discussed here are the much smaller moundlike features (each about a city block in size) scattered about in the bottom left quarter of the image. Beyond their common size, and relative isolation they display a high degree of reflectivity, most of them casting shadows that come to a point.

For clarity and ease of analysis the 12 mound-like features of interest in Fig. 2 are highlighted. The surrounding terrain is relatively free of candidates classifiable as mounds; these mounds are not pre-selected from a field of many. Some that look similar are not included for certain reasons.

For example the feature between the two larger landforms (about 1:00 from the mound at the lower left quadrant) is shown in another image to be the peak of a feature that is much larger than the other mounds. (Ref. [1] describes the mound selection criteria and statistical aspects in more detail.)

Since focus will be on angular placements of these features [2], for further clarity the image is rotated so that in Fig. 3 the bottom two are horizontal. Of the twelve let us consider the five that are most isolated from the larger land masses. The angular placements of some of these mounds (in particular what we call mounds A, D, E, G, P, O) were first discussed by Hoagland [2]. The possibility of significant angular relations among features in Cydonia was first discussed by Torun [3].

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Fig. 1 A Portion of Viking Frame 35A72.



Fig. 2 Highlighted Mounds from Viking Frame 35A72.

The five mound region is magnified and the mounds are designated in Fig. 4 by the letters GEDBA. The 4 right angles (GEA, EAB, GAD, and ABD) in the relative placements of the mounds are striking.

As implied by the angle measurements of  $88.7 \pm 3.9$ ,  $35.0 \pm 1.9$ , and  $56.3 \pm 2.8$  degrees for the triangle GEA and  $90.0 \pm 3.9$ ,  $34.8 \pm 1.5$ , and  $55.2 \pm 2.4$  degrees for triangle BAE, these two triangles (Fig. 5) are not only similar but congruent right triangles within measurement errors. (The angle measurements are made on an orthorectified version of this image and are taken with respect to the centers of the mounds, whose position is uncertain within 1 pixel - about 47 meters).

Triangles GAD and ADB (Fig. 6) are also right triangles with measured angles of  $88.2 \pm 2.7$ ,  $36.6 \pm 1.7$ ,  $55.2 \pm 2.4$  and  $90.9 \pm 5.4$ ,  $36.5 \pm 2.2$ , and  $52.6 \pm 3.3$  degrees respectively. Within measurement errors these triangles are therefore not only are similar to each other but to the previous two right triangles. Altogether there are four similar right triangles among this isolated group of just five mounds.



Fig. 3 Rotated version of fig. 2.



Fig. 4 Close up of Pentad of Mounds Displaying 4 Right Angles.

In addition, as seen in Fig. 7, there is a related isosceles triangle EDA with angles of  $71.1 \pm 3.2$ ,  $55.6 \pm 2.9$ , and  $53.2 \pm 2.7$  degrees. The angles and size of this triangle, again within measurement errors, show not only that it is isosceles but is the double of the small right triangle ADB.

The above measurements were made separately for each triangle with the vertices at the respective centers of each the three mounds. Now consider what we shall call a coordinated



Fig. 5 Two congruent right triangles.



Fig. 6 Two Further Similar Right Triangles

fit. In this fit one uses the same fit point in each mound for all triangles that have one vertex in the mound. We have found that by varying those 5 common fit points that it is possible to have the four right triangles mentioned above to be similar with a high degree of precision (less than 0.2 degrees). Right triangles have angles: 90, 45+t/2, 45-t/2. This precise coordinated fit to 4



Fig. 7 Related isosceles triangle.

similar right triangles is possible only for *t* about 19.5 degrees. (One can show the mathematically unique and exact result of  $t = \arcsin(1/3)$  radians or 19.46...degrees by use of analytic geometry). This is an example of what could be called the self-replication property of this *t* value, in that for this special value the number of appearances of these triangles increases. For different values of *t* a coordinated fit would show that not only can they not be all right triangles, but they cannot be all similar. Furthermore, for this unique *t* value, triangle ADE is precisely isosceles with angles of 45+t, 45+t, 90-t.

Figure 8 shows how close the actual mound centers are to the coordinated fit points that give this ideal geometry. The small circle at the center represents the precision to which the center of the mound is determined. The mounds are not circular but their precise shape is not of immediate interest. However, the coordinated fit points that give the ideal geometry corresponding to t = 19.5 degrees are within measurement errors at the centers of the mounds.

There are other remarkable features of this pentad of mounds. SPSR member Cesar Sirvent noted that this five sided figure leads to four sets of precisely parallel lines. As shown here in Fig. 9 those pairs of lines are (GE,AB), (GA,EB), (GB,ED), and (EA,DB). This is due to GE being parallel to AB and of equal length, and EA being parallel to DB, with the length EA = 2DB. As long as these ratios are maintained one could vertically stretch or flatten the pattern and one would still have four sets of parallel lines. What makes this pentad-based set of 4 parallel lines unique, (save an up-down reflection about the line EA and/or a horizontal one about a line perpendicular to D), is the right angle GAD.

A second curious property of this pentad of mounds is that the three different sizes of the four similar right triangles are ordered by the first three prime numbers. As indicated in Figs.



Fig. 8 Precision of coordinated fit to ideal geometry.



Fig. 9 Four sets of parallel lines 1.

10a, 10b and 10c, if one takes the smallest triangle to be one unit of area, then the area of each of the two congruent middle sized ones is 2 units and that of the large one is 3 units. The area of the obtuse triangles shown is one unit as well. (As an aside note that obtuse triangles GEB and GAB are congruent).

Not only do the three sizes of the similar right triangles corre-

spond to the first three prime numbers but also as seen in Fig. 11, the next prime number 5 appears as the area of the entire five-sided pentad. As a result the pentad of mounds displays the concept of area, with a correspondence to the first 4 prime numbers.

Stepping down one dimension from areas to lengths one finds that paralleling the basic 1, 2, 3 sequence of areas is the same sequence of triangle side relative lengths. Taking the shortest side (DB) of the smallest triangle (DBA) to be 1, then the middle side (EA) of the middle sized triangle (AEG) is 2 and the longest side (GD) of the largest triangle (GAD) is 3. As Fig. 12 emphasizes, in sequence of size (triangles ADB, GAE and GAD) the three basic aspects of the sides of a right triangle (opposite, adjacent, and hypotenuse) are ordered 1, 2, 3 sequentially with their side lengths (opposite of ADB, adjacent of GAE, and hypotenuse of GAD).

This 1, 2, 3 sequence is repeated a third time in the ratios of the sides of each similar right triangle of  $\sqrt{1}$ ,  $\sqrt{2}$ ,  $\sqrt{3}$ . (since  $\sqrt{1} = 1$ , the sides of ADB are in that sequence, and the larger right triangles are similar). Beyond that, as shown in Fig. 13, all intermound distances are multiples of  $\sqrt{2}$  and/or  $\sqrt{3}$ . For example, the length GB is  $\sqrt{2}\sqrt{2}\sqrt{3} = 2\sqrt{3}$ . (As an interesting aside note that the lengths of the sides of the two congruent obtuse triangles GEB and GAB are  $\sqrt{1}\sqrt{2} = \sqrt{2}$ ,  $\sqrt{2}\sqrt{3} = \sqrt{6}$ ,  $\sqrt{3}\sqrt{4} = \sqrt{12}$ ).

Speaking geometrically and forgetting the context of surface features on the planet Mars for a moment, the origin of these tantalizing basic geometrical and prime number features lies in the fact that these five mounds are at 5 of the 8 nodal points of a special rectangle called the square root of two rectangle (Fig. 14). The  $\sqrt{2}$  rectangle is special in that bisected perpendicular to its exterior long sides, it produces two smaller replicas of itself.

The rectangles containing the triangle EAD and GED have the same proportions, but with one-half the area of the one containing the entire pentad of mounds. The only other geometrical objects having such a duplication property is the 45 degree right triangle.

Let us now consider two of the remaining 12 mounds. Interestingly, including mound P on the far left (see Fig. 15) produces the additional triangle PEG, having angles of 92.1  $\pm$  3.8, 32.1  $\pm$  1.8, and 55.8  $\pm$  2.7 degrees. These are close enough to the ideal of 90, 45-*t*/2=35.3, 45+*t*/2=53.7 (with t = 19.5 degrees) so that a precise 6 mound coordinated fit (within 0.2 degrees) can be easily obtained, giving a fifth right triangle similar to the four above and congruent to two of them. In addition there is a hint of a double sized  $\sqrt{2}$  rectangle. (Two corners of the inferred rectangle do not exhibit mounds.) This  $\sqrt{2}$  grid-like feature was discovered by McDaniel. For the ideal geometry, the enclosed area of the 6 mound hexad of mounds is 7, the next prime number after 5.

Including mound M (see Fig. 16) produces a large replica PMD of the triangle EAD as seen by the respective measured angles of 55.1, 54.7, 70.2 versus 55.6, 53.2, 71.2 degrees. Both are close enough to the ideal of, 45+t/2=53.7, 45+t/2=53.7, 90-t = 70.5 t (with *t*=19.5 degrees) so that a precise 7 mound coordinated fit (within 0.2 degrees) can be easily obtained, giving us two similar isosceles triangles and five similar right triangles, although the coordinated fit points to the ideal are not as close to the center as with the original pentad of mounds. Another interesting feature is that the angle between the long bases PD and ED of these similar isosceles triangles is precisely 19.5 degrees, the defining angle of our ideal coordinated fit.



Figs. 10 The 1, 2, 3 sequence of area units for similar right triangles.



Fig. 11 Area of 5 units for the pentad.

Stepping up in dimensions to three dimensional space, it is of interest that the similar isosceles triangles PMD and EAD have the same proportions as the triangular cross section interior to the tetrahedron (see Fig. 17). The right triangle EXA has the same proportions as the ideal 5 similar right triangles between the 7 mounds so far included [4].

Emphasis on this connection to the solid geometry of the tetrahedron still further comes from including mound O. One finds that OPQ in Fig. 18 can have a coordinated fit (with the other mounds) to that of an equilateral triangle. The surprising connection is that the ratio between its area and that of the isosceles EAD is identical to that between the external face and internal crosssectional areas of triangles of a tetrahedron. This is exact given the ideal geometry, since the  $\sqrt{2}$  rectangular grid implies that length



Fig. 12 The 1, 2, 3 sequence of lengths for similar right triangle sides.

PG=ED. (As an aside there is a further connection between the tetrahedron and the  $\sqrt{2}$  rectangular. If the cross-section EAD is divided into two equal right triangles and moved to share a common hypotenuse, then the resultant figure is the  $\sqrt{2}$  rectangle.) Indeed, McDaniel has shown elsewhere [4] that the entire internal geometry of the tetrahedron is represented in the two-dimensional geometry of the  $\sqrt{2}$  rectangle.

## 3. STATISTICAL STUDY OF THE TWELVE MOUND CONFIGURATION

Including all twelve mounds in a coordinated fit one finds that



Fig. 13 Intermound distances as multiples of  $\sqrt{2}$  and  $\sqrt{3}$ .



Fig. 14 Relation of pentad of mounds to "2 rectangle.

altogether a coordinated fit can produce a maximum of 19 of these ideal related right and isosceles triangles. (See [1] for a detailed enumeration of these triangles and the other mounds, shown here in fig. 19). The coordinated fit to the ideal geometry yields 7 similar isosceles with angles 90-t, 45+t/2, 45+t/2 and 12 similar or congruent right triangles with 90, 45-t/2, 45+t/2. The ideal geometry corresponds to  $t=\arcsin(1/2)$ 



Fig. 15 Mound P and extended rectangular grid.



Fig. 16 Mound M with isosceles MPD similar to ADE.

3)=19.5.degrees. Are there other geometries (corresponding to coordinated fits with t=0,0.5,1.0,1.5,...,(19.5),..90) which the Cydonia mounds favor so strikingly? How do the number of appearances of these Cydonia triangles or ones with different t compare with those of randomly generated mounds over a similar area? Imagine flipping 12 coins and having them land in an area with dimensions proportional to the above figure. For each set of random landing, one records for a given t value the maximum number of similar isosceles and related right triangles obtained from a coordinated fit. For each t one would find a reliable average if enough "flips of the coins" were done. One then constructs a plot corresponding to all t values (at  $\frac{1}{2}$  degree intervals). (For the details of how this was done by computer see ref. [1]).

This plot shown in fig. 20 [5] shows how the average distri-



Fig. 17 The isosceles triangular cross section of the tetrahedron.



Fig. 18 Mound O and related equilateral triangle.

bution would look and how would it compare for each t to that generated by the Cydonia mounds and answers both of the above questions. It shows that within 0.2 degrees of precision the geometry characterized by t=19.5 degrees (the one discussed for the pentad of mounds) stands out well above other tvalues. It also shows that for the randomly generated mounds the average shows no special geometry that has an unusual peak. The self-replication property is not manifested in the plot corresponding to the randomly generated mounds. In contrast it does appear to be with the sharp peak, corresponding to the actual Cydonia mounds. However, the self-replication property cannot work unless the additional mounds are correctly placed. For example, if one had started with the tetrad of mounds



Fig. 19 Letter designations for 12 mounds.

GEAD and for the fifth mound chosen P instead of B, then there would be three instead of four similar right triangles with the ideal geometry.

In spite of this spike and the exotic prime number and geometrical properties of the mounds, could chance offer a more mundane explanation? After all there are 220 triangles between 12 mounds. Reference 1 proposed what the authors called the Random Geology Hypothesis: Given the large number of possible triangles, the finite area of each mound for the coordinated fit point to wander around in to produce the maximum number of appearances of these related right and isosceles triangles, reasonable odds for the appearance of the observed 19 related triangles may be plausible. (This is not the same as saying that geological forces are random, but that for this test, their effects over the distance of a few kilometers between the mounds are essentially random in terms of determining the locations of the mounds).

This plot focuses on the t=19.5 degree geometry, giving the number of random appearances N in 1,000,000 simulations for N ranging from 0 to 25. The plot (Fig. 21) [5] (from [1]) shows that with 6 being the most likely number of appearances, an appearance of 19 of these similar right and isosceles triangles would be an extreme outlier.

Let us examine some of the details that the above curve presents. For the 12 Cydonia mounds, the coordinated fit point was on average about 3.45 pixels from the center (with error of 1 pixel) for the t=19.5 ideal geometry. The number of random throws that resulted in 19 or more appearances with t=19.5 degrees and with this degree of precision was at a level of significance (p value) of about 0.0000155 [1], about 1/1000 of the common choice of 0.01 used to reject the null hypothesis.

There are two related critiques of this study. The first is by astrophysicist Peter Sturrock [6]: "One should not use the same data set to search for a pattern and to test for that pattern." In other words he would be criticizing our inclusion of the pentad of mounds among the 12 in our statistical study of chance appearances of right and isosceles triangles related to the angle 19.5 degree. Our reply is that the statistical analysis of the 12-mound configuration does not constitute a "search for a



pattern" but is an analysis, following the discovery of a pattern. The sequential order of the mental processes which one uses in analyzing the data has no bearing on the statistical significance of the overall pattern. The distinction between the discovery and analysis process that such a critique implies can only be artificially imposed in this case. In [1], for pedagogical purposes we showed the pattern using 6 mounds. In our original work we noticed a pattern with just the four mounds AEDG. In this paper we began with 5 mounds. In our actual discovery process we followed the pattern through all 12 mounds before deciding that a statistical analysis is called for. Where does one cutoff the data set in the discovery process? One could even argue that a pattern of sorts related to this special angle was found before our studies of mounds. We could have used earlier emphasis by Torun and Hoagland [2] who identified a possible

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reference to Tetrahedral Geometry (geometry related to the angle t=19.5 degrees) elsewhere in the area. In that case there could be a priori expectations that this geometry would appear among other features in the Cydonia area, such as the mounds. One could start with mound EAD (as we did in fact in a preliminary version of [1]) and point out right away the connection to t=19.5 degrees. In that case all twelve mounds could be part of an a priori analysis.

Either the twelve-mound pattern is statistically significant or it is not. But does the above bell curve answer that question? A related critique has been given by the mathematician Ralph Greenberg [7] where he he introduces the idea of the selfreplication property of the triangles related to the  $\sqrt{2}$  rectangle (also discussed in [1]). In private communications he stated one should broaden the analysis of the Random Geology Hypothesis to include *all geometries*, not just t=19.5 degrees. Greenberg's proposal would include all interesting symmetries, not just triangles. However, it is our contention that the class of ones that are least rare are those related to right and isosceles triangles.

Greenberg's idea, if implemented, would determine whether or not the twelve mound pattern is statistically significant. Let us summarize the new statistical work and then contrast that with Greenberg's proposal. Our fig. 20 shows that the Cydonia mounds display a spike at 19.5 degrees, compared with the random average at that angle. Figure 21 shows that it is extremely unlikely that any of the randomly created distributions that make up that average (at 19.5 degrees) would match or exceed the number 19 of appearances among the Cydonia mounds. But it does not show if it is extremely unlikely that any geometry (not just 19.5 degrees) would match or exceed the number 19 of appearances among the Cydonia mounds. Greenberg's proposal suggests, in essence, that one construct a bell curve for each value of t and sum the result. Then one could suspect that the appearance of 19 or more related right and isosceles triangles would not be such an outlier. That is, the high number of appearances from some geometry would be vastly more likely if all geometries were included and result in an acceptable p value. So, it is suggested that for each of the 1,000,000 "flips of 12 coins" one does an exhaustive search for all interesting geometries, not just the one at 19.5 degrees.

However, the new analysis shows that with all geometries included in this way the statistical anomaly holds up. What was done was the following. For each of 1,000,000 randomly chosen sets of 12 mounds a coordinated fit was performed to determine the maximum number of related right and isosceles triangles for each value of t (0,0.5,...,89.5, 90). Out of these 1,000,000 sets of data, 30 had a number of appearances equal or greater than 19. The plurality of them (10) appeared at 19.5 degrees. The second most appearances (5) were at t = 26.5degrees (arctan(1/2)). There were 2 at 18 degrees (connected to the triangle that bisects the Golden rectangle) and 2 at 30 degrees  $(\arcsin(1/2))$  and a scattering of others for a total of 30 out of 1,000,000. The angles 26.5, 18, and 30, presumably have a similar but significantly reduced self-replicating property for producing large number ( $\geq 19$ ) appearances. This would mean that for bell curves based on other values of t, 19 would even be a more extreme outlier. Thirty out of 1 million (although significantly greater than 10) is still too small to argue that including "all geometries" gives a reasonable p value. The significant self-replicating property of tetrahedral triangles singles out this geometry (t = 19.5 degrees) as the primary contributor in the new statistical analysis. Now with statistics being an unlikely prosaic explanation of the mounds geometry, let us study this self-replication property in more detail. Perhaps the geometry itself may offer an explanation.

This plot (Fig. 22) shows that statistically there is indeed a self-replicating property and that it does stand out in randomly generated mounds. It shows for each angle t (on the horizontal axis) how many times out of one million simulations (on the vertical axis) that angle produces the maximum number of appearances of the related triangles. Thus, out of 1,000,000 runs the angle 19.5 degrees produced the maximum number in 36,000 of these runs.

In the next plot (Fig. 23), for each of the 1,000,000 simulations, and for each angle that produces the maximum



Fig. 22 Randomly generated plot shows self replicating property.

number of appearances of the related triangles the number of those appearances is tabulated. The result is summed over all one million simulations.

Thus, out of 1,000,000 runs the angle 19.5 degrees produced the maximum number of related triangle appearances about 36,000 times and as seen here the total number of related right and isosceles appearances adds up to about 290,000. However, by dividing the results of this graph (for each t) by that of the previous, one sees that there is very little difference between readings for different angles.

That average number of appearances (on the vertical axis) for the maximum performing angles (on the horizontal axis) varies from about 7.5 to 8 as seen in fig. 24. There is only a slight peak at t=19.5 degrees and one again at 30. Compare this with the average plot given in fig. 20. The average there is that of the average number of triangle appearances regardless of whether or not that t was the maximum performing angle. It is mostly flat around 6. For that average there is not even a small peak in the average. The self-replication property no doubt is responsible for some of the 19 repeated triangles (due, however, to the special locations of the mounds to take advantage of this property). However, on average, that property is not significant for randomly generated mound placements.

### 4. FURTHER AVENUES FOR FUTURE ANALYSIS

First is an examination of the difference between the Pentad coordinated fit, which as seen in fig. 8 is precise, compared to the twelve mound coordinated fit in fig. 25 which is off center. If the phenomena that is responsible for the repeated appearances of the related right and isosceles triangles favors a central location for the vertices on the mounds, then this would suggest that chance is more likely to be responsible for the placements for some of the additional seven mounds other than those within the pentad than for those that make up the pentad of mounds.

Following along these lines, consider the only two mounds of the Pentad that have been resolved by the Mars Global Surveyor (MGS) satellite, mounds G and E (see Figs. 26a and 26b). These show an interesting degree of symmetry. Mound G in fig. 26a shows an axial symmetry. Mound E, in Fig. 26b shows a rare four or five-sided pyramidal shape. These symmetries plus the precision with which the Cydonia Mounds fit to the ideal geometry raises some interesting questions that could be answered by future investigations. For example, one







Fig. 24 Quotient of previous two plots.



Fig. 25 Coordinated fit points for t = 19.5 degrees.





Fig. 26 (a) High resolution MGS image of mound G., (b) High resolution MGS image of mound E.

could examine MGS images of mound-like features in areas in which there do not appear to be any significant patterns and compare their symmetry or lack thereof with that of these mounds. Is there a correlation between symmetrical placements and symmetry of shapes of the mounds on the one hand and between unsymmetrical placements and lack of symmetry of shapes of the mounds on the other?

## 5. FURTHER DISPLAYS OF FUNDAMENTAL FEATURES BY THE PENTAD

There is one further property of the pentad of mounds that is worth mentioning. The author was preparing some lecture notes on molecular quantum mechanics and was reminded that when the spin of two electrons combine to give a larger spin, "the relative orientation of the individual angular momentum (spins) are the same in all cases (the angle is about 70 degrees)" [8] (strictly speaking, never parallel). Now the opening angle EDA is 70.5 degrees for the ideal geometry. The closeness of the two values piqued the author's interest. It turns out that the angles are precisely the same. To see this, recall that the spin of the electron is quantized. Its magnitude can only be  $\hbar \sqrt{3}/2$ , while measuring the component of its spin along any direction can only give  $\pm \hbar/2$ . (The symbol  $\hbar$  is Planck's constant divided by  $2\pi$ , an elementary particle is said to have angular momentum  $j\hbar$  if the magnitude of the angular momentum vector is  $\hbar \sqrt{j(j+1)}$ . Thus, for  $j = \hbar/2$  the magnitude is  $\hbar \sqrt{3/2}$ . For  $j = 1\hbar$ , the magnitude is  $\hbar \sqrt{2}$ .). In a constant magnetic field the electron's magnetic moment (due to its spin) precesses about the field direction with the short side forming the axis of a cone. This precise, unalterable geometrical description of the electron's spin projection and magnitude is modeled exactly by the ratios of the mound separation distances corresponding to the similar right triangles. Figure 27 displays this interesting correlation. That is, the ratio of the length of DA to DB is exactly  $\sqrt{3}$ for the ideal geometry.

Now consider two electrons whose individual total spins are represented by the respective lines DA and DE in fig. 28. In that case the total angular momentum of the two-electron system has a zero projection along the line DB. It points from D perpendicular to DB with the tipped arrow. Here, not only do the individual electrons precess but their combined spin (of value  $\sqrt{2}\hbar$ ) also precesses. But the main point is that the two



Fig. 27 Ideal right triangle and electron spin.

spins combine to give the maximum possible value, and this can only occur when their relative orientation of the individual angular momentum is precisely  $\pi/2$ -arcsin(1/3)= 70.5 degrees, as occurs in the isosceles triangle ADE. This combining of ordinary spin (or rather the magnetic moments associated with spin) to give a larger spin (magnetic moment) is the microscopic origin of macroscopic magnetism. The orientation of the two sets of mounds DA and DE precisely model this spin combination in the case of the ideal geometry. (Interestingly, the 1, 2, 3 sequence accentuated earlier by areas and side lengths of triangles appears here in angular momentum sequences for the two congruent obtuse triangles of the Pentad. Referring to fig. 13, we see that triangles GEB and GAB have sides of length  $\sqrt{1(1+1)} = \sqrt{2}$ ,  $\sqrt{2(2+1)} = \sqrt{6}$ , and  $\sqrt{3(3+1)} = \sqrt{12}$ . In the theory of angular momentum each of these two triangles would describe the orientation for the addition of two angular momenta of values  $l\hbar$  and  $2\hbar$  to give  $3\hbar$ .)

### 6. CONCLUSION

There is no doubt that the ideal mathematical pentad does exhibit the basic mathematics of number and geometry and in a simple and elegant fashion. The pentad displays congruent and similar right triangles and a related isosceles. These right triangles have three different sizes with areas in the simple ratios of 1:2:3 and that of the entire pentad equal to 5. The lengths of the opposite, adjacent, and hypotenuse of the three sized triangles are 1, 2, 3 respectively. The pentad and the whole mound



Fig. 28 Ideal isosceles triangle and electron spin coupling.

structures display repeatedly aspects of a  $\sqrt{2}$  rectangular grid and the closely related geometry of a tetrahedron. The cross section of the tetrahedron is the ideal isosceles triangle of the pentad. An equilateral triangle among the mounds has the same proportions to the ideal isosceles of the pentad as that of the equilateral triangular faces of the tetrahedron to that of its cross section. From the point of view of the Cydonia mounds, the origin of all of these intriguing mathematical features of the pentad, as well as the quantum physics observations, are due to the accurate rendition (see Fig. 8) of four right angles *each* combined with an associated length ratio of the perpendicular sides of  $\sqrt{2}$ . It should be mentioned that there is no recorded instance of a geological fracture or joint intersection pattern displaying that  $\sqrt{2}$  ratio combined with a right angle.

While intriguing, the appearance of this pentad-based geometry calls out for further investigations. Are there other simple polygons whose internal geometry display similar symmetries and connections to prime numbers? If so, would there be an objective way of ranking those geometries and connections, or would their comparisons be only a matter of mathematical esthetics? Beyond that, what would the probability be of any such configuration appearing with precision by chance?

Most of the critiques by NASA and JPL geologists we consulted encouraged us to find statistical sources of the anomaly instead of geological sources. In other words, it was expected such precise geometries in geology would only be by chance. The statistical analysis of the patterns do indeed show that it is far more likely by chance to have these triangles with t=19.5 degrees having the maximum number of appearances than other

geometries. On the other hand, our statistical analysis also shows that the odds for the large number (19) of the appearances of these special triangles (or any in fact) is extremely remote. Finally we have established that the pentad of mounds display at least three basic connections to the fundamental quantum mechanics of spin angular momentum. In summary, the mound geometry, and math and physics observations may be just curiosities due to an extreme statistical fluke that gives rise to the Pentad, or alternatively an indication of some intelligent purpose behind their placement. At this stage there is no way to distinguish. However, on either alternative it would seem that further investigation of this area should be a priority, whether for geological or SETI reasons.

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